Enrollment No: _______________________ **Exam Seat No:**_______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT1 **Branch :** B.Tech (All)

Semester : 1 **Date :** 02/12/2015 **Time :** 10:30 To 1:30 **Marks :** 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (1 marks each) **(14)**

a) n^{th} derivative of $y = \dfrac{1}{x+a}$ is

(a) $\dfrac{(-1)^n n!}{(x+a)^{n+1}}$ (b) $\dfrac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ (c) $\dfrac{(-1)^n n!}{(x+a)^n}$ (d) none of these

b) If $y = \sin^{-1} x$ then x equal to

(a) $1 - \dfrac{y^2}{2!} + \dfrac{y^4}{4!} - ......$ (b) $y - \dfrac{y^3}{3!} + \dfrac{y^5}{5!} - ......$ (c) $1 + y + \dfrac{y^2}{2!} + \dfrac{y^3}{3!} + ...$

(d) none of these

c) The Maclaurin's series expansion of $log(1-x)$ is ____________________

d) What is the value of y_3? where $y = \sin 2x$

(a) $8\sin 2x$ (b) $-8\sin 2x$ (c) $-8\cos 2x$ (d) $8\cos 2x$

e) The addition of two convergent series is ______

(a) convergent (b) divergent (c) oscillatory (d) none of these

f) The series $a + ar + ar^2 + ar^3 + \cdots \infty$ is divergent if

$(a)\ |r| < 1$ $(b)\ r \geq 1$ $(c)\ r < -1$ $(d)\ r = -1$

g) State De Moivre's theorem.

h) Separate $sinh(x+iy)$ into real and imaginary parts

i) $e^{i\pi/2} =$ ______

$(a)\ 0$ $(b)\ 1$ $(c)\ i$ $(d)\ -1$

j) State Euler's theorem for homogeneous function.

k) Find $\frac{dy}{dx}$ for $x^2 + y^2 - xy = 0$

l) Discuss about symmetry of the curve $x^3 + y^3 - 3axy = 0$

m) Find the asymptote of the curve $xy^2 = 4(2 - x)$

n) $\frac{\partial(u,v)}{\partial(x,y)} \cdot \frac{\partial(x,y)}{\partial(u,v)} =$ ______

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

A If $u = \log\left(x^3 + y^3 + z^3 - 3xyz\right)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$ **(05)**

B State the Euler's theorem on homogeneous function and use it to prove that **(05)**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \tan^3 u, \text{ where } u = \sin^{-1}\left(\sqrt{x^2 + y^2}\right).$$

C Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x}{2}\right)^{\frac{1}{x}}$ **(04)**

Q-3 Attempt all questions

A Test for convergence of the series $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$ **(05)**

B If $y = a\cos(\log x) + b\sin(\log x)$ then prove that **(05)**

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

C If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, find $\frac{\partial^2 z}{\partial x \partial y}$. **(04)**

Q-4 Attempt all questions

A Test the convergence of the series $\frac{1^3}{3} + \frac{2^3}{3^2} + \frac{3^3}{3^3} + \frac{4^3}{3^4} + \ldots\ldots$ **(05)**

B Prove that $(1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8 = -(2)^8$ **(05)**

	C	If $x = e^v \operatorname{cosec} u$, $y = e^v \cot u$, find $\dfrac{\partial(x,y)}{\partial(u,v)}$	**(04)**
Q-5	**Attempt all questions**		
	A	Trace the curve $y^2(2a-x)=x^3$.	**(05)**
	B	If α and β are roots of equation $z^2-2\sqrt{3}z+4=0$ then prove that $\alpha^n+\beta^n = 2^{n+1}\cos\frac{n\pi}{6}$	**(05)**
	C	Find the n[th] derivative of $y=\log\left(x+\sqrt{1+x^2}\right)$.	**(04)**
Q-6	**Attempt all questions**		
	A	Trace the curve $r=a(1+\cos\theta)$.	**(05)**
	B	Solve $x^7 + x^4 + i(x^3+1) = 0$ using De-Moiver's theorem.	**(05)**
	C	Expand $\tan^{-1}x$ in powers of $\left(x-\frac{\pi}{4}\right)$.	**(04)**
Q-7	**Attempt all questions**		
	A	Prove that $\sinh^{-1}x = \log(x+\sqrt{x^2+1})$	**(05)**
	B	Express $sin^8\theta$ in a series of cosines of multiples of θ.	**(05)**
	C	Find $\dfrac{dy}{dx}$, if $\sin(xy)=e^{xy}+x^2y$	**04)**
Q-8	**Attempt all questions**		
	A	Examine for extreme values for the function $x^2+y^2+6x+12$	**(05)**
	B	Separate into real and imaginary parts $\sqrt{i}^{\sqrt{i}}$	**(05)**
	C	Using Taylor's series, arrange $x^3 - 3x^2 + 4x + 3$ in power of $(x-2)$.	**(04)**